Standard Cosmological Evolution in f(R) Model to Kaluza Klein Cosmology

[a]**A. Aghmohammadi**[1], [b,c]**Kh. Saaidi**[2], [a]**M. R. Abolhassani**[3], [b]**A. Vajdi**[4]

[a]*Plasma Physics Research Center, Science and Research Branch Islamic Azad University of Tehran, Iran*

[b]*Department of Physics, Faculty of Science, University of Kurdistan, Pasdaran Ave., Sanandaj, Iran*

[c]*Faculty of Science, Azad University of Sanandaj, Sanandaj, Iran*

Abstract

In this paper, using $f(R)$ theory of gravity we explicitly calculate cosmological evolution in the presence of a perfect fluid source in four- and five-dimensional, spacetime in which this cosmological evolution in self-creation is presented by Reddy et al 2009 Int. J. Theor. Phys. 48 10. An exact cosmological model is presented using a relation between Einsteins gravity field equation components due to a metric with the same component from $f(R)$ theory of gravity. Some physics and kinematical properties of the model are also discussed.

Keywords: Kaluza Klein line element; f(R) Gravity; Modified gravity

[1] agha35484@yahoo.com
[2] ksaaidi@uok.ac.ir
[3] mrhasani@modares.ac.ir
[4] Avajdi@uok.ac.ir

1 Introductions

In recent years, there has been a considerable interest in alternative theories of gravitation. The observation that universe appears to be accelerating at present times has caused one of the greatest problem to modern cosmology. High precision data from type Ia supernova, the cosmic microwave background and large scale structure seem to hint that the universe is presently dominated by an unknown form of energy, dubbed dark energy [1, 2, 3, 4, 5, 6]. One obvious contender for the role of dark energy is Einstein's cosmological constant, but particle physics failed to predict the correct density. We refer the reader to [7]

Recently, a modification of general relativity itself was suggested to explain this accelerating universe [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. For review see e.g [19, 20, 21]. The assumption is that the Ricci scalar of the Einstein -Hilbert action is replaced by add a perturbation function $h(R)$ to the Einstein - Hilbert action[22]. Recently, it has been claimed that, in all theories which behave as a power of R at large or small R, standard cosmological evolution can not be obtained [23, 24]. These models have raised much recent interest, due to their perhaps simple nature. The presence of ghosts and stabilities have been studied [25, 26, 27, 28].

Study of higher -dimensional models are also important because of the underlying idea that the cosmos at its early stage of evolution might have had a higher dimensional era [31]. The extra space reduce to a volume with the passage of time which is beyond the ability of experimental observation at the moment [31]. Our motivation to consider the $f(R)$ model in the five dimensional space times in the presence of a perfect fluid, is that, the same problem is considered in the self-creation cosmology(SCC), in which corresponding Mach's Principle(MP) is incorporated in SCC by assuming the inertial masses of fundamental particles are dependent upon their interaction with a scalar field ϕ coupled to the large scale distribution of matter in a similar fashion as Brans Dicke theory (BD). We refer the reader for review see e.g [29, 31, 30]. However, instead, with recourse to the f (R) model, perhaps because of their simple nature, and on Q3 modifying part of geometry, instead of matter in the equation of Einstein, we obtain the same result. In this paper, we have obtained a cosmological evolution in the presence of perfect fluid source represents disordered radiation in four and five dimensional space time[31]. However, the cosmological evolution in five dimensional, in models $f(R)$ in the presence of perfect fluid source have not been investigated.

2 Theoretical Framework

For convenience, we consider a class of modified gravity in which we add a perturbative function $\epsilon h(R)$, to the Einstein-Hilbert action, where ϵ is a small parameter. We consider an action as

$$l1S = \int \sqrt{-g}\left[\frac{R + \epsilon h(R)}{2} + kL_m\right] d^4x. \tag{1}$$

Here R is a Ricci scalar and L_m is matter Lagrangian. The field equation, using the metric approach, can be derived from action as,

$$G_{\mu\nu} = -\epsilon\left[G_{\mu\nu} + g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu + \frac{g_{\mu\nu}}{2}\left(R - \frac{h(R)}{\varphi(R)}\right)\right]\varphi(R) + kT_{\mu\nu}, l2 \tag{2}$$

where $G_{\mu\nu}$ and $T_{\mu\nu}$ are Einstein and stress-energy tensors respectively, $\Box \equiv \nabla_\alpha\nabla^\alpha$ and $\varphi(R) = dh(R)/dR$. Although this theory can be written as scaler tensor theory [14, 32], we shall not use the conformal transformation, we complete all our calculations in the Jordan frame, given by the action above. We consider the metric given by

$$l3ds^2 = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2). \tag{3}$$

In which the space-time is assumed to be of flat Friedmann -Robertson-Walker (FRW), the components of Ricci tensor can be written in terms of the scale factor, $a(t)$, as

$$R^t_t = 3\frac{\ddot{a}}{a}, l4 \tag{4}$$

$$R^r_r = R^\theta_\theta = R^\phi_\phi = \frac{a\ddot{a} + 2\dot{a}^2}{a^2}, l5 \tag{5}$$

where the over dot denotes differentiation with respect to cosmic time. The components of Einstein tensor in terms of the scale factor, $a(t)$, in the flat Friedmann-Robertson-Walker line element, can be find as following

$$G^t_t = 3\left(\frac{\dot{a}}{a}\right)^2, l6 \tag{6}$$

$$G^r_r = G^\theta_\theta = G^\phi_\phi = -(\frac{2a\ddot{a} + \dot{a}^2}{a^2}). l7 \tag{7}$$

The components of d'Alembertian in the flat Friedmann-Robertson-Walker line element are as following

$$\begin{aligned}
\nabla_t \nabla^t &= \frac{\partial^2}{\partial t^2}, l8 \qquad (8)\\
\Box &= \frac{\partial^2}{\partial t^2} + \frac{3\dot{a}}{a}\frac{\partial}{\partial t}, l9 \qquad (9)\\
\nabla_r \nabla^r = \nabla_\theta \nabla^\theta &= \nabla_\phi \nabla^\phi = \frac{\dot{a}}{a}\frac{\partial}{\partial t}. l10 \qquad (10)
\end{aligned}$$

The energy-momentum tensor, $T_{\mu\nu}$, for a perfect fluid distribution is given by

$$l11 T_{\mu\nu} = (p + \rho)\, u_\mu u_\nu - p g_{\mu\nu}. \qquad (11)$$

Here, p is the isotropic pressure, ρ the energy density and u_μ represents the four velocity of the fluid. Corresponding to the line element given by (**??**) the four velocity vector u_μ satisfies the equation

$$l12 g_{\mu\nu} u^\mu u^\nu = 1. \qquad (12)$$

In a co-moving coordinate system the components of Einstein tensor with the help of field equation, using the metric approach, can be derived from action (**??**). In the field equation (**??**), if $h(R)/\varphi(R) = 0$ as $R \to 0$ i.e.;

$$l13 \lim_{R\to 0} \frac{h(R)}{\varphi(R)} \to 0, \qquad (13)$$

we can neglect $h(R)/\varphi(R)$, and then we have

$$\begin{aligned}
G^\nu_\mu &= -\epsilon \left(\delta^\nu_\mu \Box - \nabla^\nu \nabla_\mu\right) \varphi(R) + T^\nu_\mu, l14 \qquad (14)\\
G^t_t &= -\epsilon \left(3\frac{\dot{a}}{a}\frac{\partial}{\partial t}\right) \varphi + k\rho, l15 \qquad (15)\\
G^r_r = G^\theta_\theta = G^\phi_\phi &= -\epsilon \left(\frac{\partial^2}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial}{\partial t}\right) \varphi(R) + kp. l16 \qquad (16)
\end{aligned}$$

Where ρ and p are components of T^ν_μ. Furthermore, from contracting field equation (**??**), we can find

$$l17 R = 3\epsilon \Box \varphi, \qquad (17)$$

from combination equations (**??**,**??**) with (**??**,**??**), one obtains

$$3\left(\frac{\dot{a}}{a}\right)^2 = -\epsilon\left(\frac{3\dot{a}}{a}\frac{\partial}{\partial t}\right)\varphi + k\rho, l18 \tag{18}$$

$$\frac{2a\ddot{a}+\dot{a}^2}{a^2} = \epsilon\left(\frac{\partial^2}{\partial t^2}+2\frac{\dot{a}}{a}\frac{\partial}{\partial t}\right)\varphi - kp.l19 \tag{19}$$

The field equations (**??**,**??**,**??**) are three independent equation in four unknown variable a, φ, ρ, p. Hence to get a determinate solution and correspondence with the standard Einstein cosmology solution one has to assume a physical or mathematical condition. We choose the scale factor as

$$l20a(t) = t^{\gamma}, \tag{20}$$

and the equation of state as

$$l21\rho = 3p, \tag{21}$$

which represents disordered radiation in four dimensional space. Now, using equations (**??**,**??**) the field equations (**??**,**??**,**??**) yields an exact solution for a, φ, ρ, p. According to equations (**??**, **??**) and (**??**), we can choose φ as :

$$l22\varphi = \alpha \ln t + c, \tag{22}$$

where α and c are arbitrary constants. Substituting equations (**??**, **??**, **??**), into equation (**??**), power-law solution can be identified as

$$l23\gamma = \frac{1}{2} + \frac{\alpha\epsilon}{4}. \tag{23}$$

By substituting equation (**??**) into equation (**??**,**??**), we can arrive at

$$\rho = \frac{3}{4kt^2}\left(1+\alpha\epsilon\right), l24 \tag{24}$$

$$p = \frac{1}{4kt^2}\left(1+\alpha\epsilon\right), l25 \tag{25}$$

$$H = \frac{2+\alpha\epsilon}{4t}, \qquad l26 \tag{26}$$

$$q > 0l27, \tag{27}$$

which with $\epsilon = 0$, solutions above are correspond with epochs of radiation domination. The energy density ρ, the isotropic pressure p, tend to zero as time increases indefinitely. Also Hubble's parameter, H, tend to zero as $t \to \infty$. The positive value of the deceleration parameter, q, shows that the model decelerates in the standard way.

3 Kaluza-Klein Cosmological in f(R) Theory of Gravity

Once more we consider action (**??**) together with field equation (**??**) that yields to variation of action (**??**) for the line element of five dimensional 5D Kaluza-Klein space time given by

$$l28ds^2 = dt^2 - a^2(t)\left(dx^2 + dy^2 + dz^2\right) - A^2(t)d\psi^2. \tag{28}$$

In Kaluza-Klein space time, the components of Ricci tensor and Ricci scaler can be written in terms of the scale factor, $a(t)$ and $A(t)$. As a result, we find

$$\begin{aligned} R_{tt} &= \frac{3\ddot{a}}{a} + \frac{\ddot{A}}{A}, l29 && (29)\\ R_{\psi\psi} &= -\frac{3A\dot{a}\dot{A}}{a} - A\ddot{A}, l30 && (30)\\ R_{xx} = R_{yy} &= R_{zz} = -2\left(\dot{a}\right)^2 - \frac{a\dot{a}\dot{A}}{A} - a\ddot{a}, l31 && (31)\\ R &= 6\left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right) + 6\frac{\dot{a}\dot{A}}{aA} + 2\frac{\ddot{A}}{A}.l32 && (32) \end{aligned}$$

The components of Einstein tensor in terms of the scale factors $a(t)$ and $A(t)$, in the Kaluza Klein space-time line element, can be found as following :

$$\begin{aligned} G_t^t &= 3\left(\frac{\dot{a}}{a}\right)^2 + 3\frac{\dot{a}\dot{A}}{aA}, l33 && (33)\\ G_\psi^\psi &= -3\left(\frac{\dot{a}}{a}\right)^2 - 3\frac{\ddot{a}}{a}, l34 && (34)\\ G_x^x &= -\left(\frac{\dot{a}}{a}\right)^2 - 2\frac{\ddot{a}}{a} - 2\frac{\dot{a}\dot{A}}{aA} - \frac{\ddot{A}}{A}.l35 && (35) \end{aligned}$$

Dolambrian and its components to the Kaluza-Klein line element are as following :

$$\begin{aligned}
\nabla_t \nabla^t &= \frac{\partial^2}{\partial t^2}, l36 && (36)\\
\nabla_\psi \nabla^\psi &= \frac{\dot{A}}{A}\frac{\partial}{\partial t}, l37 && (37)\\
\nabla_x \nabla^x = \nabla_y \nabla^y &= \nabla_z \nabla^z = \frac{\dot{a}}{a}\frac{\partial}{\partial t}, l38 && (38)\\
\Box &= \frac{\partial^2}{\partial t^2} + \left(\frac{3\dot{a}}{a} + \frac{\dot{A}}{A}\right)\frac{\partial}{\partial t}.l39 && (39)
\end{aligned}$$

In the Kaluza-Klein space time and a co-moving coordinate system the components of Einstein tensor with the help of field equations (**??**, **??**, **??**) are given from variation of action (**??**) as following

$$\begin{aligned}
G_t^t &= -\epsilon\left(\frac{3\dot{a}}{a} + \frac{\dot{A}}{A}\right)\frac{\partial}{\partial t}\varphi(R) + k\rho, l40 && (40)\\
G_\psi^\psi &= -\epsilon\left(\frac{\partial^2}{\partial t^2} + \frac{3\dot{a}}{a}\frac{\partial}{\partial t}\right)\varphi(R) + kp, l41 && (41)\\
G_x^x = G_y^y = G_z^z &= -\epsilon\left(\frac{\partial^2}{\partial t^2} + \left(\frac{2\dot{a}}{a} + \frac{\dot{A}}{A}\right)\frac{\partial}{\partial t}\right)\varphi(R) + kp.l42 && (42)
\end{aligned}$$

Where ρ and p are component of the T^μ_ν. Furthermore, from contraction field equation (**??**), we can obtain as:

$$l43R = \frac{8}{3}\epsilon\left(\frac{\partial^2}{\partial t^2} + \left(\frac{3\dot{a}}{a} + \frac{\dot{A}}{A}\right)\frac{\partial}{\partial t}\right)\varphi. \tag{43}$$

From equations (**??**-**??**) we obtains:

$$\begin{aligned}
3\left(\frac{\dot{a}^2}{a^2} + \frac{\dot{a}\dot{A}}{aA}\right) &= -\epsilon\left(\frac{3\dot{a}}{a} + \frac{\dot{A}}{A}\right)\frac{\partial}{\partial t}\varphi(R) + k\rho, l44 && (44)\\
\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a} + 2\frac{\dot{a}\dot{A}}{aA} + \frac{\ddot{A}}{A} &= \epsilon\left(\frac{\partial^2}{\partial t^2} + \left(\frac{2\dot{a}}{a} + \frac{\dot{A}}{A}\right)\frac{\partial}{\partial t}\right)\varphi(R) - kp, l45 && (45)\\
+3\frac{\dot{a}^2}{a^2} + 3\frac{\ddot{a}}{a} &= \epsilon\left(\frac{\partial^2}{\partial t^2} + \frac{3\dot{a}}{a}\frac{\partial}{\partial t}\right)\varphi(R) - kp.l46 && (46)
\end{aligned}$$

The set field equations (**??**-**??**) are four independent equation in five unknowns ρ, p, a, A, φ. Hence to get a determinate solution one has to assume a physical or mathematical condition. We solve the above set of field equations with the equation of state trace $T = 0$

$$l47\rho = 4p, \tag{47}$$

which is analogous to the equation of state before $\rho = 3p$ which represents disordered radiation in four dimensional space. To obtain a determinate solution we also use a relation between the metric potentials as

$$l48A = \xi a^n. \tag{48}$$

Where ξ, n are constants. Substituting equations (**??**, **??**, **??**) into (**??**, **??**, **??**),we can obtain γ, and the other unknowns parameters for a small ϵ, as following

$$\gamma = \frac{3+n}{\eta}\left(1 + \frac{2}{3}\zeta\alpha\epsilon + \dots\right), l49 \tag{49}$$

$$A = \xi t^{n\frac{3+n}{\eta}\left(1+\frac{2}{3}\zeta\alpha\epsilon+\dots\right)}, l50 \tag{50}$$

$$H = \frac{1}{t}\frac{3+n}{\eta}\left(1 + \frac{2}{3}\zeta\alpha\epsilon + \dots\right), l51 \tag{51}$$

$$q = -\frac{(\gamma-1)}{\gamma}, l52 \tag{52}$$

$$\rho = \frac{(3+n)^2}{kt^2\eta}\left(\frac{3(n+1)}{\eta} + (1 + \frac{4(n+1)\zeta}{\eta})\alpha\epsilon + \dots\right), l53 \tag{53}$$

$$p = \frac{(3+n)^2}{4kt^2\eta}\left(\frac{3(n+1)}{\eta} + (1 + \frac{4(n+1)\zeta}{\eta})\alpha\epsilon + \dots\right), l54 \tag{54}$$

where

$$\eta = n^2 + 3n + 6, \tag{55}$$

$$\zeta = 1 - \frac{n^2+1}{(n+3)^2}. \tag{56}$$

We require that the model decelerates in the standard way, so that q must be bigger than zero. Therefore, we find an extra condition on γ as

$$l550 < \gamma < 1, \tag{57}$$

and then

$$l55n \neq -3. \tag{58}$$

For considering the stability of this model of $f(R)$, we must obtain the second derivation of $f(R)$ with respect to R. It is well known that, for an arbitrary $f(R)$ model, if $\frac{d^2f(R)}{dR^2} > 0$, then that model is stable [25, 28]. Therefore, using equations (**??**, **??**), we can obtain

$$l56\frac{d^2f(R)}{dR^2} = \epsilon\frac{d\varphi}{dR} = \frac{3t^2}{16\epsilon((n+3)\gamma-1)}. \tag{59}$$

Hence, the requirement for stability of our model, is that for $\epsilon > 0$, n must satisfy

$$l57n > \frac{1}{\gamma} - 3, \tag{60}$$

and for the case $\epsilon < 0$, n must satisfy

$$l58n < \frac{1}{\gamma} - 3 \tag{61}$$

Also, for considering the singularity of this $f(R)$ model of gravity at initial time, we need the Ricci scalar as a function of time. Therefore, with combining equations, (**??**, **??**,**??**, **??**) and (**??**), one can obtain the Ricci scalar as following

$$l59R = \frac{8}{3}\epsilon\alpha\left(-1+(3+n)\gamma\right)\frac{1}{t^2}. \tag{62}$$

It is clearly seen that, $R = \infty$ as t tends to zero, i.e.;

$$\lim_{t\to 0} R \to \infty, \tag{63}$$

this means that, this model has initial singularity. Thus, the five dimensional Kaluza-Klein cosmological line element corresponding to the above solutions can be written in $\epsilon = 0$ by:

$$l60ds^2 = dt^2 - \left(t^{\frac{2n+6}{6+3n+n^2}}\right)\left(dx^2+dy^2+dz^2\right) - \xi^2 t^{n\left(\frac{2n+6}{6+3n+n^2}\right)}d\phi^2 \tag{64}$$

4 Some Physical Properties Of the Model

Equation (**??**$\cdots$**??**) represents an exact five dimensional Kaluza-Klein cosmological in the framework of $f(R)$ theory of gravity in the presence of perfect fluid source. We observe that the $f(R)$ theory of gravity has initial singularity, (**??**), and we have seen that with n defined by equations (**??**,**??**), the requirement of stability is satisfied. For the Eq(**??**), the physical and kinematical variables which are important in cosmology are ρ, p, q, H the

energy density ρ, the isotropic pressure p, tend to zero as time increases indefinitely. For this model Hubble parameter tend to zero as $t \to \infty$, the positive value of the deceleration parameter q, shows that to the both four and five dimension decelerates in the standard way.

5 Discussion

A modification of gravity has been suggested in the form of $R + \epsilon h(R)$ theories, in the presence of perfect fluid source. The five dimensional Kaluza-Klein and four dimensional line element in the $f(R)$ theory of gravity has stability and initial singularity. The model is expanding, non-rotating and decelerates in the standard way.

References

[1] A. G. Riess *et al.*, Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999).

[2] J. L. Tonry *et al.*, Astrophys. J. **594**, 1 (2003).

[3] C. L. Bennett *et al.*, Astrophys. J. Suppl, **148**, 1 (2003).

[4] C. B. Netterfield *et al.*, Astrophys. J. **571**, 604 (2002).

[5] N. W. Halverson *et al.*, Astrophys. J. **568**, 38 (2002)

[6] C. Deffayet *et al.*, Phys. Rev. **D65**, 044023 (2002).

[7] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phy. **D 15**, 1753 (2006).

[8] S. Capozziello *et al.*, Int. J. Mod. Phys. **D 11**, 483 (2002).

[9] K. Freese and M. Lewis, Phys. Lett. **B540**, 1 (2002).

[10] G. Dvali and M. S. Turner, Astro. Phy. 0201033 (2002).

[11] S. Nojiri, S. D. Odintsov, Gen. Rel. Grav. **36**, 1765 (2004).

[12] N. Arkani-Hamed, H-C. Cheng, M. A. Luty and S. Mukohyama, JHEP **05**, 043528 (2004).

[13] S. Capozziello, S. Carloni, and A. Troisi, RecentRes.Dev.Astron. Astrophys. **1**, 625 (2003).

[14] S. Nojiri and S. D. Odintsov, Phys. Rev. **D 68**, 123512 (2003).

[15] S. M. Carroll, Phy. Rev. **D 71**, 063513 (2005).

[16] M. C. B. Abdalla, S. D. Odintsov, Class. Qunt. Grav. **22**, L35 (2005).

[17] S. Capozziello, S. Nojiri, S. D. Odintsov and A. Troisi, Phys. Lett. **B 639**, 135 (2006).

[18] S. A. Appleby and R. A. Battye, Phy. Lett. **B 654**, 7 (2007), .

[19] D. A. Easson, Int. Mod. phys. **A 19**, 5343 (2004).

[20] S. Nojiri and S. D. Odintsov, Int. J. Geom. Meth. Mod. Phys. **4**, 115 (2007).

[21] M. Trodden, Int. J. Mod. Phys. **D 16**, 2065 (2008).

[22] J. D. Evans and L. M. H. Hall and P. Caillo, Phys. Rev. **D 77**, 083514 (2008).

[23] L. Amendola, D. Polarski and S. Tsujikawa, Phys. Rev. Lett. **98**, 131302 (2007).

[24] L. Amendola, D. Polarski and S. Tsujikawa, Int. J. Mod. Phys. **D**, 1555 (2007).

[25] A. D. Dolgov, M. Kawasaki, Phys. Lett. **B 573**, 1 (2003).

[26] R. Dick, Gen. Rel. Grav. **36**, 217(2004).

[27] A. D. Felice, M. Hindmarsh, and M. Trodden, JCAP **0608**, 005 (2006).

[28] V. Faraoni, phys. Rev. **D72**, 124005 (2005).

[29] G. A. Barber, gr-qc/0302088v3, 12 (2005).

[30] G. A. Barber, astro-ph/0401136v6, 12 (2005).

[31] D. R. K. Reddy *et al.*, Int. J. Teor. Phys. **48**, 10 (2009).

[32] E. E. Flanagan, Class. Quant. Grav. **21**, 417 (2003).